PERGAMON

International Journal of Solids and Structures 36 (1999) 5665–5666

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Authors' Closure

Reply to comments on "Dual analysis for path integrals and bounds for crack parameter"

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In the comments on the paper by Wu et al. (1998), Maigre made the argument that $I^* = I$. This is contradictory to our analytical and numerical results. After reading the letter we found a fundamental mistake that stems from a misconception of the 'exact differential'.

Recalling the development of J-integral as the potential energy release rate $-\mathrm{d}\pi/\mathrm{d}a$ for a given crack system as detailed by Rice (1968), all quantities involved in the energy π are taken as a function of the coordinates x_j and the crack length a. Thus

$$u_i = u_i(x_j, a), \quad \sigma_{ij} = \sigma_{ij}(x_j, a), \ldots \tag{1}$$

The above function relationships are still valid for the integrals I^* in Wu et al. (1998) and I in Bui (1974), since they were developed as the complementary energy release rate $\mathrm{d}\pi_c/\mathrm{d}a$.

As has been shown by Wu et al. (1998),

$$I^* - I = \int_\Gamma \frac{\partial(u_i\sigma_{i2})}{\partial x_j}\mathrm{d}x_j \tag{2}$$

which in view of eqn (1), $(u_i\sigma_{i2})$ is also a function of x_j and a. Thus, the exact differential of concern takes the form of

$$\mathrm{d}(u_i\sigma_{i2}) = \frac{\partial(u_i\sigma_{i2})}{\partial x_j}\mathrm{d}x_j + \frac{\partial(u_i\sigma_{i2})}{\partial a}\mathrm{d}a \tag{3}$$

but is never in the form of

$$\mathrm{d}(u_i\sigma_{i2}) = \frac{\partial(u_i\sigma_{i2})}{\partial x_j}\mathrm{d}x_j. \tag{4}$$

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0020-7683/99/$ - see front matter
PII: S0020-7683(98)00242-X

Obviously, the commenter's argument that "the integral in (2) is an exact differential" is false. Therefore, the final conclusion is $I^* \neq I$.

References

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